Safer Ski Jump Landing Surface Design Limits Normal Impact Velocity

ABSTRACT: Skiing and snowboarding have become more acrobatic with terrain park jumps and other manmade features playing a prominent role in an increase in serious spinal cord injuries. Yet these jumps rarely, if ever, involve formal or detailed design or engineering. This paper presents a coherent theory and methodology for the design of ski jump landing surfaces. The motion of the skier center of mass is modeled. Although jumpers may land in many configurations, we assume that the probability of severe landing injury is correlated with the component of skier velocity perpendicular (normal) to the landing surface \(v\), or, more understandably, with the corresponding equivalent fall height \(EFH\). The requirement that \(v\) and \(EFH\) be small is satisfied by making the landing surface nearly parallel to the skier flight path at landing. Safer landing surfaces that limit \(EFH\) to a given value are shown to satisfy a first order ordinary differential equation (ODE). Having chosen an \(EFH\) deemed safe enough by the designer, integration of this ODE provides members of an infinite family of landing surfaces that limit the \(EFH\) to the desired value, for any jumper in-run velocity. Using the takeoff ramp angle as another design variable, one can choose a member of this family to fit on almost any available jump site. The formulation incorporates the fact that skiers can slightly modify velocity direction and magnitude at takeoff by jumping and is valid for any landing body configuration. Such landing surfaces can still yield exhilarating flight experiences with relatively large flight times and air height above the surface, but without the danger posed by jumps created in an ad hoc manner, which can expose the skier to much larger unsafe equivalent fall heights.

KEYWORDS: snowboard, safety, terrain parks, spinal cord injury, equivalent fall height

Introduction

Over the last decade the world skiing industry has changed markedly. Increases in the proportion of snowboarders have led to development of terrain parks, and acrobatic skiing/snowboarding now plays a larger overall role in recreational skiing. To accommodate typically younger and more adventurous sliders, many terrain park features now involve jumps, but little scientific effort has been spent understanding their safe design and use.

The evolution in the kinds of use at ski resorts has led to a corresponding evolution in the types of injuries experienced [1,2]. According to Shealy et al. [3], the snowboard injury rate doubled during the period 1990–2000 from 3.37 to 6.97 per 1000 skier days. Before 1990 most injuries involved the lower extremity, but since then there has been a significant increase in upper extremity, head, and spinal cord injuries (SCI) [4]. The SCI rate is estimated to be about 1 per \(10^6\) skier days [5] for the general skiing population, but it has been estimated to be as high as 40 per \(10^6\) days for “elite” skiers/snowboarders who perform more acrobatic and energetic maneuvers. Skiing has now replaced American football as the second leading cause of sports SCIs in the United States [6]. Furthermore, although these rates can be argued to be low on an absolute basis, the economic and social costs of SCIs are very high [6] since they can be permanently debilitating.

Historically, design of skiing equipment (skis, snowboards, bindings, helmets, etc.) has been handled by credentialed company design engineers. The creation of terrain park features, however, is most often a local activity at each ski resort. Although they are usually tested and modified before they are first opened to the public, little if any scientific analysis and design occurs before the construction of ski/snowboard terrain park jump landing surface shapes. More often than not they are simply fabricated with a snow cat based on consideration of available resources and the past experience of the driver/builder in construction of jumps.

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Such a traditional “tabletop” jump is shown in Fig. 1. The jumper approaches from a takeoff ramp on the left, is airborne over the roughly horizontal tabletop, and hopefully lands on the nearly constant down-slope to the right. The severity of landing varies substantially with takeoff velocity and landing position on such a jump and can be large enough to be unsafe.

This paper presents a coherent theory and methodology for the design and use of safer ski jump landing surface shapes. It shows how these shapes can be designed and fabricated to limit the exposure of the jumper to large normal impact forces on landing.

**Methods**

Although aerodynamic lift and drag forces can have important effects on the trajectories in longer flights (e.g., Refs. [7–9]), for simplicity we first neglect these aerodynamic forces (as well as wind) and make the approximation that the only force acting on the jumper is that due to gravity. Aerodynamic forces can and should be incorporated in the numerical analysis of longer flights.

Similarly we neglect the deformations and rotations of the jumper and consider only the motion of the center of mass. Anytime a jumper is airborne for a long period, control of body landing orientation can be lost, resulting in impact with the surface by portions of the jumper’s body other than the skis. As a result of the approximation above, the analysis to follow cannot and does not attempt to deal with this possibility. The term “safe” is used below only in the context of landing surface shape design and fabrication to limit the exposure of the jumper to large normal velocities on landing. This is the part of jumper “safety” that can be controlled by the landing surface shape designer and the analysis below details how this can be accomplished.

We use an $xy$ coordinate system with $x$ horizontal, $y$ vertical, and origin at the takeoff point (Fig. 2) to characterize two functions; the jumper’s center of mass flight path $y(x) + L$ (where $L$ is the leg length) and the shape of the landing slope $y_s(x)$. It is assumed that the jumper approaches the takeoff point with in-run velocity $v_i$ at takeoff ramp angle $\beta_o$ from the horizontal. In many, if not most, cases the jumper merely slides off the ramp and the takeoff velocity vector $v_o$ is parallel to the ramp so that $v_{xo} = v_i \cos \beta_o$ and $v_{yo} = v_i \sin \beta_o$.

It is possible, however, for the jumper to significantly augment the velocity perpendicular to the ramp by an amount $\Delta v$ (Fig. 3) and this possibility must be accounted for. An upper limit for this “jump” is given by $\Delta v = (2gd)^{1/2}$, where $d$ is the maximum vertical height the jumper able to raise his/her center of mass by jumping in a 1 g gravity field wearing boots and boards/skis on the feet. This is subject to ankle extension limitations from the bindings and the additional board or ski mass. For world class Nordic ski jumping athletes [7,10] the height $d$ is in the range $0 < d < 0.4$ m, but for more typical recreational skiers or snowboarders a reasonable upper bound is probably considerably smaller ($d = 0.25$ m), corresponding to $\Delta v = 2.21$ m·s$^{-1}$.
This $\Delta v$ changes the takeoff angle from the ramp angle by $\Delta \theta = \tan^{-1}(\Delta v/v_i)$ so that the takeoff angle $\theta_o = \beta_o + \Delta \theta$ and the actual jumper takeoff velocity is given by $v_o = (v_i^2 + \Delta v^2)^{1/2}$. In the case where the jumper does not jump at takeoff, but rather merely coasts off the takeoff ramp, the direction of the initial velocity vector is parallel to the takeoff ramp and $\theta_o = \beta_o$ and $v_o = v_i$. Although the jumper can similarly decrease the takeoff angle [11] this is not of interest here, since it is larger takeoff angles that result in larger potential impact forces to be protected against. Note that, even if $\Delta v$ is assumed to be constant, the actual maximum takeoff angle $\theta_o$ is a function of jumper velocity $v_i$.

In the case of negligible aerodynamic forces, the velocity components along the path after flight time $t$ are given approximately by the projectile equations $v_x = v_{x_0} = v_o \cos \theta_o$ and $v_y = v_{y_0} - gt = v_o \sin \theta_o - gt$ and the positions by $x = v_{x_0} t$ and $y = v_{y_0} t - gt^2/2$.

Eliminating $t$ allows $y$ to be expressed as the familiar parabola in terms of only $x$ and the parameters $v_o$ and $\theta_o$ as
Inverting Eq 1 gives an analytic expression for the initial velocity \( v_o \) required to reach a given \( x, y \),

\[
v_o = \sqrt{\frac{x^2 g}{2(x \tan \theta_o - y) \cos^2 \theta_o}} .
\]

(2)

and, at the point \( xy \) on the path, the speed \( v \) and the angle of the flight path measured from horizontal \( \theta = \tan^{-1}(dy/dx) \), respectively, are also given as functions of \( x \) by

\[
v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_o^2 \cos^2 \theta_o + (v_o \sin \theta_o - gx/v_o \cos \theta_o)^2},
\]

(3)

\[
\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(\tan \theta_o - gx/v_o \cos \theta_o).
\]

(4)

What goes up must come down. During landing the skier velocity vector is changed to become parallel to the slope. Because the coefficient of friction between snow and skis or clothing is typically small, the velocity component parallel to the slope can remain essentially constant, but the perpendicular component \( v_\perp \) is brought to zero by large snow contact forces. This occurs more slowly when the landing occurs on the skis and the legs act to cushion the fall, or possibly more rapidly when, after loss of body orientation, contact occurs on other parts of the skier’s body.

In either case, impact severity and the potential for injury is naturally measured by the magnitude of the normal velocity \( v_\perp \) when the jumper first possibly touches the surface (i.e. reaches one leg length \( L \) above the surface). For example, nonjumping falling skiers often hit the ground at high tangential velocities and rarely have severe injuries, while skiers with severe injuries often have hit the ground with high normal (perpendicular) velocity components. The more nearly parallel to the slope the velocity vector is at landing, the smaller its perpendicular component \( v_\perp \), and the less normal impulse is incurred, providing a safer landing with less risk for injury. This is the essential idea on which this paper is based.

Because velocity is often not intuitively understood, a simpler but comparable impact severity measure is equivalent fall height (EFH), defined as the fall height \( h = v_\perp^2/2g \) in a 1 g environment that results in velocity \( v_\perp \). EFH has been previously called “equivalent landing height” and used to characterize ski jumping impacts in the analysis of Nordic ski jump landing slope safety [8]. It is also used in governmental safety measures and standards for parachutes and aircraft [12,13].

EFH can be shown to have three parts (\( \text{EFH} = h_1 + h_2 + h_3 \)); \( h_1 = v_o^2 \sin^2 \theta_o / g \) is the maximum height of the skier’s path above takeoff; \( h_2 \) is the vertical distance of the impact point below the takeoff point (Fig. 2). Although \( h_1 \) and \( h_2 \) have simple geometric interpretations, the third component \( h_3 \) does not. It describes the dependence of EFH on the landing surface slope. When impact occurs on a horizontal landing slope, \( h_3 = 0 \) and \( h = h_1 + h_2 \). More generally, \( h_3 \) is positive or negative, when the landing surface slope \( dy/dx \) is positive or negative, respectively. When the landing slope is nearly parallel to the jumper’s path at landing, \( h_3 < 0 \) and can nearly completely cancel the contributions \( h_1 \) and \( h_2 \), resulting in a low-impact landing with less risk for injury.

Now the fundamental requirement is that the velocity \( v_\perp \) perpendicular to the slope at impact be made small enough to correspond to a suitably small equivalent fall height \( h \),

\[
v_\perp = \sqrt{2gh} = v \sin(\phi - \theta).
\]

(5)

As is clear from Eq 5, this can be done by choosing the angle \( \phi \) of the landing surface to be close enough to the flight path angle \( \theta \) at landing.

The safe jump design problem posed here can be stated as follows: What should the snow surface shape \( y_s(x) \) be, for a given takeoff ramp angle \( \beta_o \), to subject the skier to a given limited (and presumably safe) EFH \( h \) for any jumper velocity \( v_j \)? This design constraint results in a differential equation for the snow surface \( y_s(x) \), whose solution indeed limits EFH to the specified value, in the following way. Solving for \( \phi \) from Eq 5, substituting for \( \theta \) and \( y \) from Eqs 4 and 1, and using the fact that \( \tan \phi = dy_j/dx \) allow one to write the soft landing requirement above as a first order differential equation for the landing slope.
\[
\frac{dy_s}{dx} = \tan \left\{ \tan^{-1} \left( \tan \theta_o - \frac{gx}{v_o^2 \cos^2 \theta_o} \right) + \sin^{-1} \left( \frac{2gh}{v_o^2 \cos^2 \theta_o + v_o \sin \theta - \frac{gx}{v_o \sin \theta}} \right) \right\}.
\] (6)

For a solution \( y_s(x) \) to the differential equation above, the slope of the landing surface \( y'_s(x) \) must be a monotonically decreasing function of \( x \), since the larger the jumper velocity \( v_o \), the larger the velocity \( v \) at impact and the more steeply the landing slope must be inclined to result in the same EFH. If the surface \( y_s(x) \) begins at \( x=0 \) with a positive slope, at some \( x \) it must therefore become horizontal at its highest point. For the trajectory (corresponding to some in-run velocity \( v_i \) that impacts at this highest surface point, the term \( h_3=0 \) and the equivalent fall height is (neglecting drag) simply the sum of the heights \( h = h_1 + h_2 \), the height difference between the trajectory zenith and the landing surface zenith. At values of \( x \) less and greater than that of this landing surface zenith, the landing surface slope is positive and negative, respectively. Said another way, given a ramp angle and in-run jumper velocity \( v_o \), a landing surface corresponding to a given fall height \( h \) has its zenith at the point along the trajectory that lies exactly the distance \( h \) below the trajectory zenith. Thus, the landing surface zenith allows an easy parameterization of the family of landing surfaces with a given fall height \( h \), since it is possible to integrate the slope differential equation forward and backward from this zenith.

How small an EFH \( h \) is safe enough? Since the center of mass height of normal standing adults is of the order of 1 m, treating the motion of the skier center of mass underestimates the potential EFH by about this amount. Humans have evolved to tolerate, to some extent, falls in 1 g of a meter or two. On the other hand, few humans can be expected to bear equivalent fall heights of 4–10 m (especially in a possibly compromised body orientation at landing in which a substantial portion of the impulse might result in spinal bending loads and consequent spinal cord injury). Although the ultimate choice of \( h \) must be left up to the designer, a value in the neighborhood of \( h=1 \) m seems reasonable and is here used as an example.

Additional design flexibility exists. The specified EFH \( h \) need not be constant along the landing slope. For example, if the landing surface zenith is above the takeoff point, any convex smooth surface slope shape between takeoff and the zenith that arrives at the zenith horizontally will cause a gradual taper of equivalent fall height EFH from zero at takeoff to the specified \( h \) at the point of intersection. Whether the surface zenith is above or below takeoff, a particularly straightforward way to provide such an initial taper uses a straight line from the takeoff point to the point of tangency with the landing surface.

An algorithm to calculate a sample of the family of landing surfaces for a given EFH \( h \) (over a range of jumper in-run velocities \( v_i \) and for a given ramp angle \( \beta_o \)) is:

1. Choose a fall height \( h \) and takeoff ramp angle \( \beta_o \).
2. Choose a range of in-run velocities \( v_{i\text{min}} \leq v_i \leq v_{i\text{max}} \).
3. At each velocity \( v_i \):
   a. calculate the maximum jumper velocity \( v_{oi} \) and takeoff angle \( \theta_{oi} \),
   b. calculate the zenith of the trajectory and the zenith of the landing surface, the point on the jumper path a distance \( h \) below the trajectory zenith,
   c. integrate the surface differential Eq 6 forward and backward from the landing surface zenith to fill in the ascending and descending portions of the surface out to the maximum velocity possible,
   d. taper to takeoff if desired.

**Results and Discussion**

Shown in blue solid (lower) lines in Fig. 4 is a sample of five (from the infinite family of) safe landing surface shapes calculated as solutions to Eq 6 for constant ramp angle \( \beta_o=25 \) deg and EFH \( h=1 \) m. Also shown is a family of eight jumper paths corresponding to eight evenly spaced in-run velocities \( 5 < v_i < 40 \) mph (2.24 < \( v_i < 17.88 \) m·s\(^{-1}\) ). Every landing from every (black) flight path onto every (blue) landing slope occurs with the same equivalent fall height \( h=1 \) m corresponding to a component of velocity perpendicular to the slope of \( v_\perp = 4.43 \) m·s\(^{-1}\). Thus, all the landing surfaces in Fig. 4 are equally safe in this context.

Note that the safer surface solutions designed to prevent skier exposure to EFH > \( h \) do not resemble the classically fabricated tabletop jump shown in Fig. 1 (which is roughly approximated by two straight
line segments; the horizontal tabletop and the constant slope landing area. On the contrary, the safer surface solutions have continuously and monotonically decreasing slopes as a function of $x$.

Another obvious feature of the subset of five surfaces is that they apparently asymptotically approach a common origin and asymptote nearly (but not exactly) parallel to the takeoff ramp at small values of $x$. Furthermore, the infinite family of safe landing surfaces is ubiquitous. The entire $xy$ half-plane roughly below and to the right of the asymptote is filled densely with solutions. (An infinite number exist both between each of the five landing surfaces shown in Fig. 4 and outside these as well.) The lower landing surfaces have lower zeniths that occur at smaller values of $x$, and they have smaller (larger negative) slopes at any value of $x$ than the higher surfaces do. On the same jumper trajectory (corresponding to the same takeoff velocity) impact occurs on the lower landing surfaces at larger values of $x$ and at a larger negative landing slope angle than on the higher landing surfaces (but with identical EFHs!). These steep landing slopes on the lower surfaces provide a natural limitation for their use since it becomes increasingly impractical to envision landing on a slope steeper than, say, $\phi = 30–35$ deg. Yet, as discussed below, when there is enough space able to be devoted to the jump, it is always possible to find a surface shape that limits EFH to the design value $h$ by decreasing the takeoff angle $\beta_o$.

A denser, more complete set of landing surfaces similar to that shown in Fig. 4 can provide a design template for actual jump design and fabrication planning. A landing surface design algorithm is portrayed in Fig. 5. The first step for the designer is to decide what EFH $h$ is adequately safe. Then suppose some cross sectional area $(x_L,y_L)$ bounds the space available for the landing surface placement. Thus far a given takeoff ramp angle $\beta_o$ has been assumed, but this parameter can also be used in the design process. If $\beta_o$ can be freely chosen, it is always possible to select $\beta_o$ and to calculate a corresponding safe landing surface that fits within the available area $(x_L,y_L)$ and accommodates the highest conceivable jumper in-run velocity. If, for reasons other than safety, a specific $\beta_o$ is deemed necessary, then a landing surface from the corresponding set can be chosen that fits within the available region, but the jumper path that intersects this surface at the lower right-hand corner $(x_L,y_L)$ may correspond to an in-run velocity that is too small. In this case the landing surface is safe, in the sense that it restricts EFH to the design value $h$, only up to that velocity. In this case, control of jumper velocity will be necessary to insure that jumpers use the jump within the safe range of velocities.

Thus far we have shown that it is possible to design jump landing surfaces that limit the EFH (and the corresponding impulse normal to the landing surface) to an arbitrarily low value. But will these jumps be
any fun? To answer this we must first consider the related question: Why do jumpers jump? The attraction is certainly the exhilaration felt from long flight times far above the snow surface in near zero-g that also provide the opportunity for acrobatic maneuvers. It is natural to ask whether this exhilaration is still present on the safer landing slopes discussed here.

For the same five landing surface solutions previously shown in Fig. 4, we calculated two measures of exhilaration; flight time and jump air height. Both are plotted in Fig. 6 as a function of total jump length, measured along the landing surface. Although the groupings of points show the individual dependences on both in-run velocity and the vertical position of the landing surfaces, Fig. 6 shows that, roughly speaking, both these exhilaration measures are a function of only the distance jumped. Nevertheless, seemingly considerable exhilaration can be achieved while maintaining adequate safety since, for example, a flight time and air height of roughly 2 s and 3 m, respectively, can be achieved on jumps of...
about 30 m. However, this does require an in-run velocity of 35 mph (15.8 m·s⁻¹), 11% higher than the velocity of 31.5 mph (14.2 m·s⁻¹) measured as the mean of the top speeds for intermediate skiers with good visibility [14].

Although Fig. 6 shows that flight time and jump air height depend mostly on jump distance, it is probably more exciting to land on a steeper slope than on a flatter one. Thus, a third exhilaration measure might be the steepness of the landing slope itself. This might be a reason to choose such a slope from the family of possible surface shapes that all have the same EHF.

Using the safer landing surface shapes discussed here will not guarantee that injuries cannot occur in jumps, but it will greatly reduce the likelihood of such injuries by reducing the component of velocity perpendicular to the slope on impact. Any time that jump flight durations in the neighborhood of 2 s occur, there is the possibility of loss of control of body orientation and subsequent snow impact in a compromised body position. Nevertheless, the smaller the equivalent fall height, the smaller the probability of serious injury resulting from impacts normal to the snow surface.

How might the present results be used by the skiing industry to aid in the construction of safer jump landing slopes? For each fall height h a one page report of a representative family of landing slopes (yₑ versus x) could be provided in tabular form. The ski slope operator could then choose a desired equivalent fall height h and one of the shapes from those provided on which to base the actual fabrication. Few high tech tools would be required: a measuring tape, laser level and measuring rod would suffice. The landing surface shapes would obviously need to be maintained in the face of additional snow fall, melt and distortion through jumper use.

Conclusions

Ski jump landing surfaces can be designed and managed to prevent skier exposure to unsafe equivalent fall heights. An explicit design methodology has been presented that results in a landing surface shape yₐ(x) for a relatively safe landing. Although the emphasis here has been primarily on design of landing slopes for terrain park jumps at ski resorts, the theory is equally applicable to Nordic jumping at jump distances approaching 100 m, although such efforts must include aerodynamic lift and drag forces in the design calculations. A relatively straightforward extension of the present theory allows this.

Acknowledgment

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References


JAI101630, “Safer Ski Jump Landing Surface Design Limits Normal Impact Velocity”, published online in the January 2009 issue of JAI.

Author: Mont Hubbard

This article was published online in the January 2009 issue of JAI and in a special technical publication, STP1510, Skiing Trauma and Safety: 17th Volume. The author has informed ASTM of errors in the published paper. The descriptions of the errors and their locations are as follows:

1. (Page 4) the denominator of the equation 10 lines above Eq (5) defining $h_1$ should be $2g$. It should read

$$h_1 = \frac{v_o^2 \sin^2 \theta_o}{2g}$$

2. (Page 4) five lines below Eq (5) should read "$v_i"", not "$v_j""

3. (Page 5) the last two terms in the denominator of the radical in Eq (6) should be included in parentheses and squared. Eq (6) should read

$$\frac{dy}{dx} = \tan \left[ \tan^{-1} \left( \tan \theta_o - \frac{gx}{v_o^2 \cos^2 \theta_o} \right) \right] + \sin^{-1} \left( \frac{2gh}{\sqrt{v_o^2 \cos^2 \theta_o + \left( v_o \sin \theta - \frac{gx}{v_o \sin \theta} \right)^2}} \right)$$

The Errata will be cross-referenced online to this paper. ASTM apologizes for the errors and regrets any inconvenience to its readers.

Sincerely,

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